Efficient 3-D Ultrasound Imaging From Cylindrical Apertures

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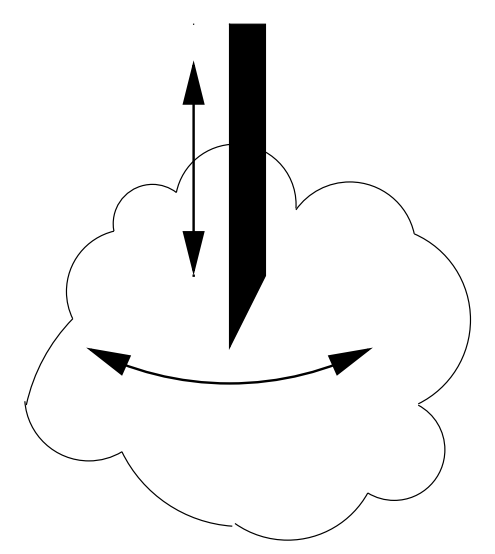
ABSTRACT

A number of new imaging modalities collect data from cylindrical platforms. *In vivo* imaging needles and intravascular ultrasound imaging catheters are examples of this geometry, where imager rotation and translation parallel to the cylinder axis are the only allowed motions. Efficient three-dimensional ultrasound image formation in these cases can be challenging when the aperture is small and/or highly curved. A frequency-domain imaging algorithm is obtained by approximating the free-space point spread function in cylindrical coordinates and obtaining its Fourier transform by analogy with the equivalent problem in Cartesian coordinates. We further propose an effective use of limited aperture by placing a focused transducer across the aperture, thereby creating a virtual source at the focus which is treated as a real, unfocused source by the imaging algorithm. This approach retains the simplicity and potential angular resolution of a small single element, yet permits full use of the available probe aperture and a higher energy output. Computer simulations and experimental ultrasonic results with wire targets show that this imaging technique attains the theoretical resolution dictated by the operating wavelength and transducer characteristics. (Supported by NIH CA 079179)

1 Introduction

- This work was motivated in part by the ongoing development of ultrasonic microprobes—tiny, high-frequency ultrasound transducers fabricated on the tips of needles and designed for *in vivo* operation. As the operating frequency increases beyond 100 MHz, these probes may eventually resolve details at the cellular level and aid the diagnosis of tumors.
- Such a probe can only be moved in two ways—translation in and out of the tissue, and rotation about the probe axis. This suggests the use of a cylindrical coordinate system.
- To obtain good resolution *in vivo*, a 3-D reconstruction must be performed, yet the imaging aperture is extremely limited.
- Another application with similar constraints: intra-vascular ultrasound imaging (IVUS).



2 Synthetic aperture imaging in a cylindrical geometry

- Time-domain beamforming (a.k.a. delay-and-sum, hyperbola summation) is simple to implement for any geometry [1], but slow, especially in 3-D.
- Frequency-domain image formation is more efficient, thanks to the FFT.
- A frequency-domain method was recently published for 2-D circular geometries; the necessary Fourier transform pair was derived using the *principle of stationary phase* approximation. [2]
- Here, we first obtain the 3-D point spread function via scalar diffraction theory, then make a paraxial approximation to obtain the needed Fourier transform pair.

3 3-D point spread function

The Rayleigh-Sommerfeld diffraction formula,

$$U(P_0) = \int\!\!\int_{\Sigma} \frac{1}{j\lambda} U(P_1) \frac{e^{jkr_{01}}}{r_{01}} \cos(\vec{n}, \vec{r}_{01})\, ds$$

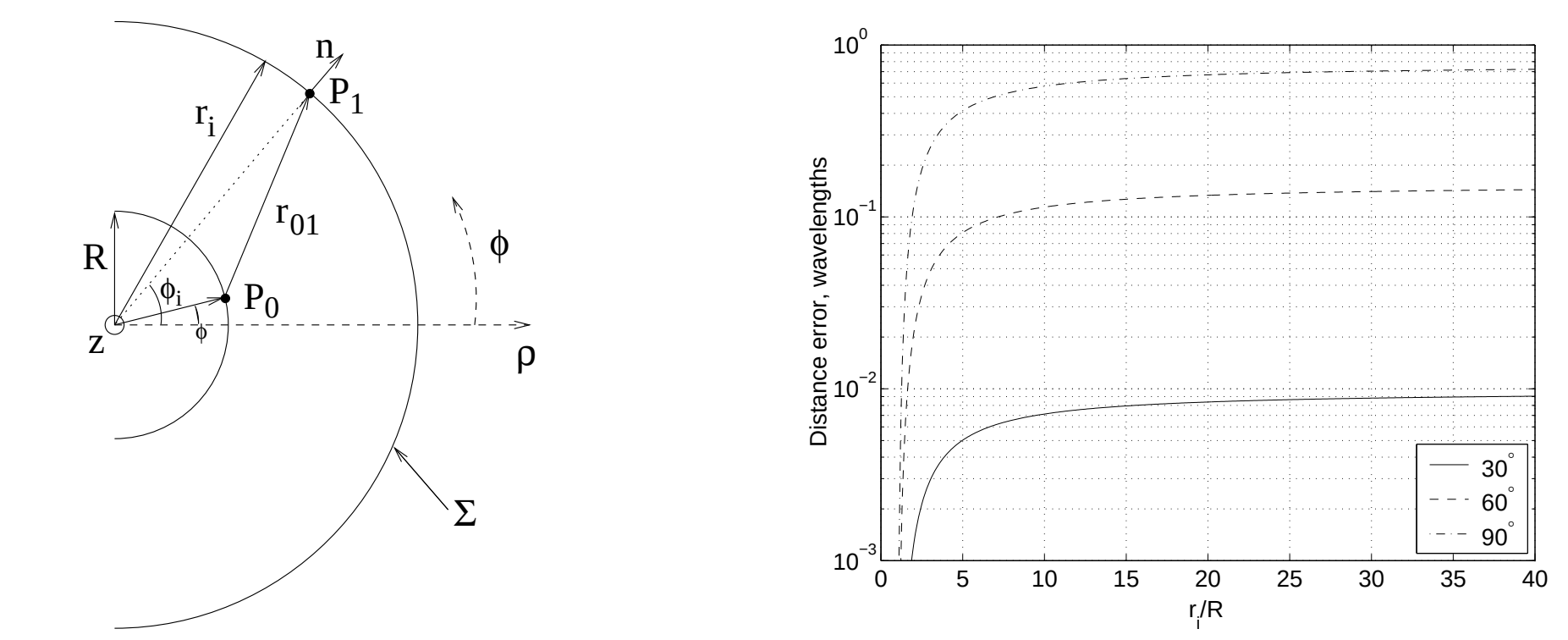
relates the field measured on an inner cylinder to a source distribution on an outer cylinder (see picture, below). Using cylindrical coordinates, we obtain a spatial convolution integral with the point spread function

$$h(\phi, z) = \frac{-j}{\lambda} \cdot \frac{(R_i - R\cos\phi) R_i \exp(jk\sqrt{R^2 + R_i^2 - 2RR_i\cos\phi + z^2})}{R^2 + R_i^2 - 2RR_i\cos\phi + z^2}.$$

If the transducer beamwidth is not too wide,

- $\cos\phi \approx 1 - \phi^2/2$ in the distance expressions (r_{01} in the original formula)
- $\cos\phi \approx 1$ elsewhere

How accurate is this approximation? The following graph illustrates, for a typical experiment, the distance error in wavelengths vs. the ratio of the target and recording surface radii. Clearly, the paraxial approximation holds up well even for beamwidths approaching 60 degrees.



Left: Cylindrical coordinate system for derivation of 3-D PSF.
Right: Error in the distance function due to the paraxial approximation.

With the approximation, we have

$$\Rightarrow h(\phi, z) \approx \frac{-j}{\lambda} \cdot \frac{(R_i - R) R_i \exp(jk\sqrt{(R - R_i)^2 + RR_i\phi^2 + z^2})}{(R - R_i)^2 + RR_i\phi^2 + z^2}.$$

This has the same form as the Huygens-Fresnel free-space PSF in Cartesian coordinates, which has a well-known spatial Fourier transform. Thus, the desired transform follows easily from the scaling property:

$$H(f_\phi, f_z) \approx \sqrt{\frac{R_i}{R}} \exp\left(j2\pi(R_i - R) \sqrt{\frac{1}{\lambda^2} - \frac{f_\phi^2}{RR_i} - f_z^2} \right)$$

4 Image formation algorithm

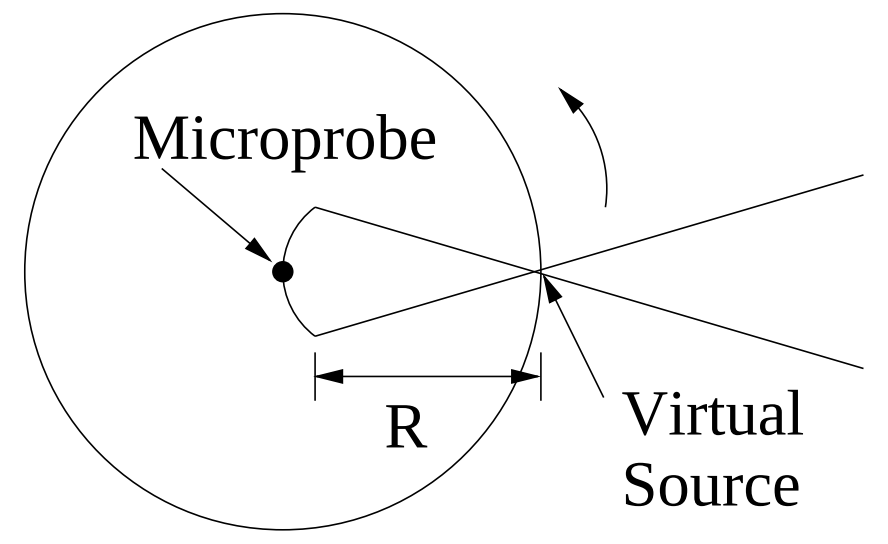
For monostatic (zero-offset) data collected on a cylindrical aperture, the "exploding reflectors" model can be used to image targets on deeper, concentric cylindrical surfaces (within the limits of the paraxial approximation). *H* (with λ = *c*/2*f*) is used to back-propagate the wavefield. Following Busse, [3]

1. Take the 3-D FFT of the raw data: $(\phi, z, t) \longrightarrow (f_\phi, f_z, f)$
2. Multiply the (f_ϕ, f_z) planes at each *f* by $H(f_\phi, f_z)$ for the target depth
3. Average over temporal frequency *f*
4. Take the inverse 2-D FFT

(In geophysics this is known as *phase shift migration* [4].)
Note that *H* is a function, not only of the relative distance between concentric surfaces, but also of their absolute radii. For this reason, a full Stolt-like migration procedure is not possible.

5 Using aperture effectively

- A well-known result in synthetic aperture imaging states that when the effective transducer/antenna diameter is *D*, the limit on lateral resolution is approximately *D*/2 [5].
- To achieve diffraction-limited resolution at high-frequency ultrasonic wavelengths, this suggests a very small transducer—much smaller even than the diameter of the needle! This would be difficult to build and incapable of radiating sufficient energy.
- Idea: Use a virtual source element [6]. In this approach a mechanically focused transducer is placed across the entire needle diameter, creating a virtual source at the focus. This focus is treated as a source of diverging spherical waves, so the surface traced out by the focus becomes the [virtual] recording surface used for imaging.
- This has the advantage of allowing a higher transmit power, while preserving near-diffraction-limited resolution if the focal ratio (f/number) of the transducer is small.



6 Lateral resolution

The lateral resolution may be determined from the extent of the data collected in the spatial frequency domain. This in turn is governed by the phase modulation imparted to the echos as the transducer steps across the aperture. For the virtual source technique with a transducer focal ratio of *F*, and using the Rayleigh criterion,

$$\delta_\phi \approx \frac{F\lambda}{2R}\ \text{rad} \quad = \quad \frac{F\lambda r_i}{2R}\ \text{m at depth } r_i$$
$$\delta_z \approx \frac{F\lambda}{2}\ \text{m}$$

References

[1] M. O'Donnell, "Efficient synthetic aperture imaging from a circular aperture with possible application to catheter-based imaging," *IEEE Trans. UFFC*, vol. 39, no. 3, pp. 366-380, May 1992.

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[3] L.J. Busse, "Three-dimensional imaging using a frequency-domain synthetic aperture focusing technique," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 39, no. 2, pp. 174-179, March 1992.

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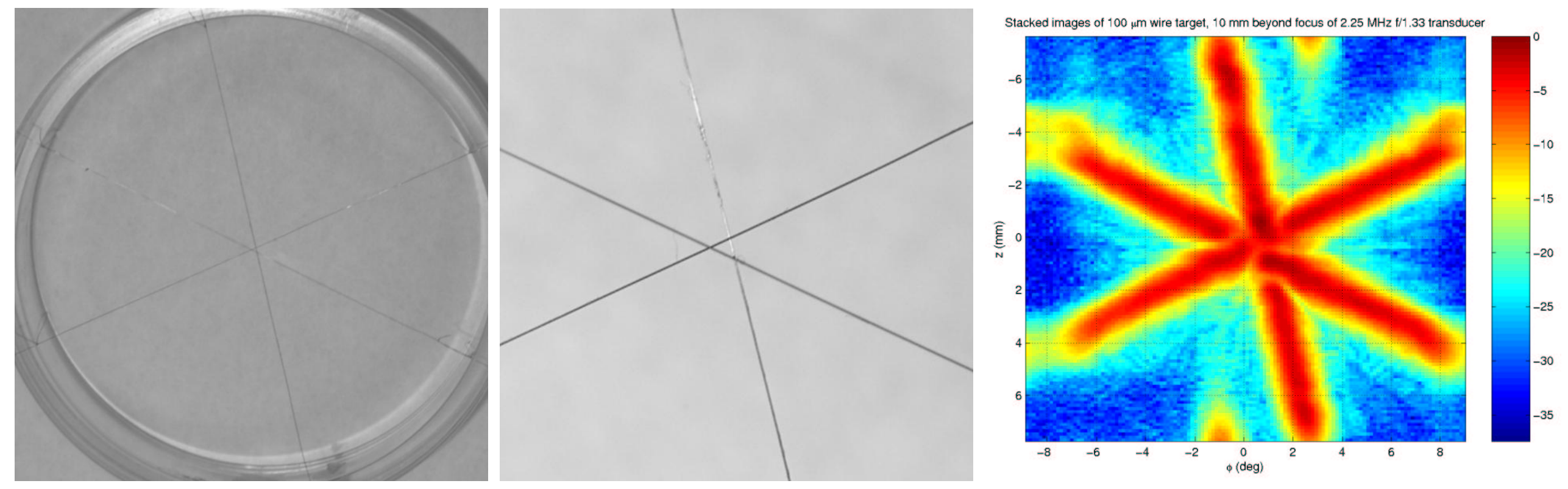
[5] P.T. Gough and D.W. Hawkins, "Unified framework for modern synthetic aperture imaging algorithms," *International Journal of Imaging Systems and Technology*, vol. 8, no. 4, pp. 343-358, 1997.

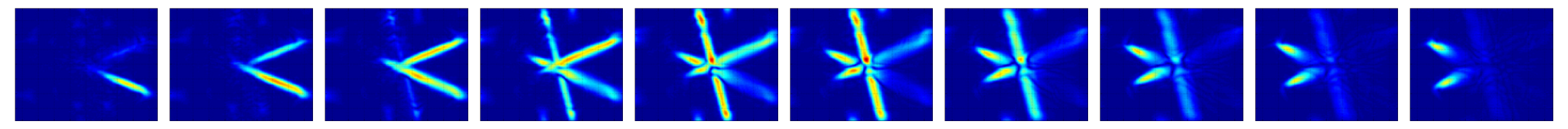
[6] C.H. Frazier and W.D. O'Brien, Jr., "Synthetic aperture techniques with a virtual source element," *IEEE Trans. UFFC*, vol. 45, no. 1, pp. 196-207, January 1998.

7 Experimental results

Experiment 1: A focused, 2.25 MHz, f/1.4 transducer was mounted to a support arm and translated/rotated such that its focus scanned a 15.4-mm by 17.9-degree section of a cylindrical surface. A target of crossing 100-micron wires was placed 10 mm beyond the focus.

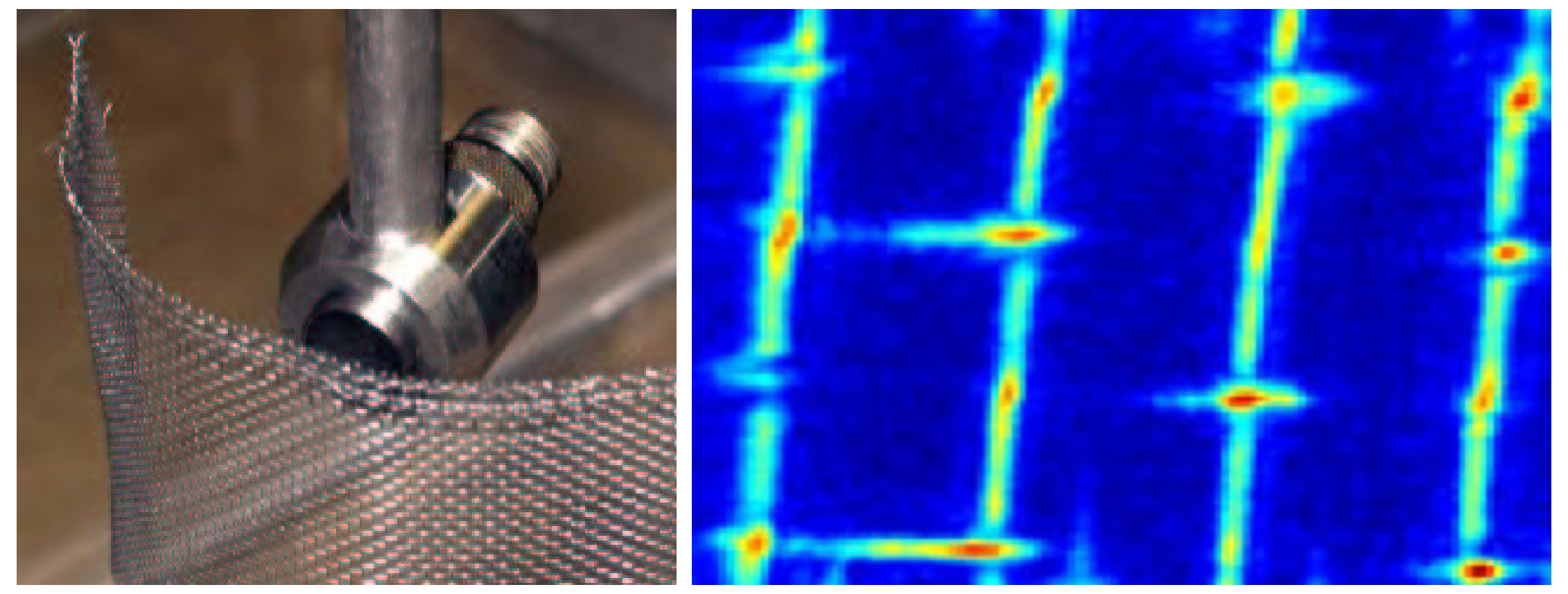


Left and center: 100-micron wire target and holder.
Right: Reconstructed images of the target, stacked over depth, log scale.



This sequence of unstacked images reveals the progression of the imaging cylinder to deeper concentric surfaces as they cut across the plane of the wire target.

Experiment 2: A focused, 15 MHz, f/1.5 transducer was mounted to a support arm and translated/rotated such that its focus scanned a 6.4-mm by 8.3-degree section of a cylindrical surface. A piece of aluminum screen from the hardware store (280-micron wire) formed the target, 5 mm beyond the focus.



Left: Experimental apparatus showing target and 15 MHz transducer.
Right: Reconstructed images stacked over depth show the weave of the screen.

The resolution achieved experimentally is only slightly worse than the resolution predicted by theory.

8 Conclusions

- Frequency domain, 3-D synthetic aperture imaging can be adapted to a cylindrical geometry, and the algorithm performs well.
- The virtual source technique enables efficient use of a limited aperture by permitting a much larger transducer than would otherwise be allowed.
- These results may find application in other imaging systems.