

## The “N-shot” Maximum-Brightness Algorithm: An Efficient and Robust Delay-Aberration Estimator

BRENT S. ROBINSON AND MARK A. HAUN  
Philips Medical Systems, Bothell, WA 98021 USA

### 1 Background

The maximum-brightness criterion originated in adaptive optics. [1] Nock et al. successfully applied it to medical ultrasound. [2]

- **Basic idea:** Adjust delay profile across the aperture to maximize the integrated brightness in a speckle-containing ROI.
- **Reasoning:** Better focus implies fewer scatterers per resolution cell, which increases the mean speckle signal. Thus, optimality is tied to smallest PSF.
- The method is simple: Only image data are required.

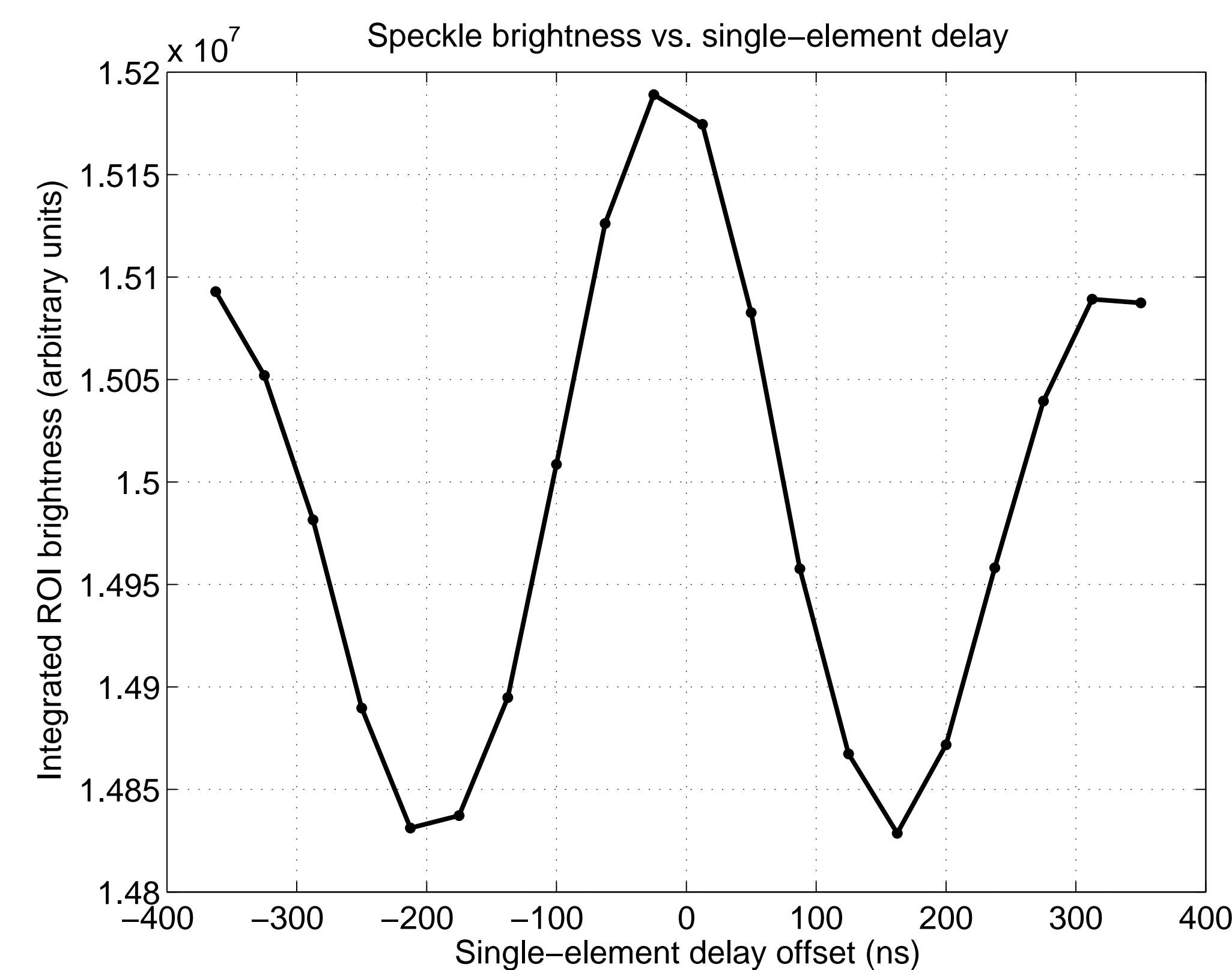
The implementation in [2] adjusts each delay using an iterative line search, stepping in small increments until the ROI brightness peaks.

This has some disadvantages:

- **Slow** (and  $\uparrow$  step size  $\Rightarrow$   $\downarrow$  delay resolution)
- **Nondeterministic** run time (don't know *a priori* how many steps)
- At reasonable step sizes, optimization endpoint is very sensitive,  $\Rightarrow$  **susceptible to false maxima** from **thermal noise** or **tissue motion-induced brightness fluctuations**.

### 2 Rationale

Observe that ROI brightness is nearly sinusoidal w.r.t. variations in the nominal delay on a single element:



That is, we can model the brightness as

$$B(\tau) = u + A \cos(2\pi f_o \tau - \phi), \quad (1)$$

where  $f_o$  is an assumed or calibrated mean frequency  
 $\tau$  is the beamformer delay offset for the element in question  
 $u$  is the baseline, or average, ROI brightness  
 $A$  and  $\phi$  are the amplitude and phase of the brightness variation.

Two ways to get  $f_o$ :

1. Calculate it from knowledge of the transmit pulse and medium characteristics.
2. Measure it from a calibration curve like the one above.

Then, only three unknowns ( $u$ ,  $A$ , and  $\phi$ ) at each array element!  
 $\Rightarrow$  **Three ROI firings (“shots”) suffice to find the optimal delay.**

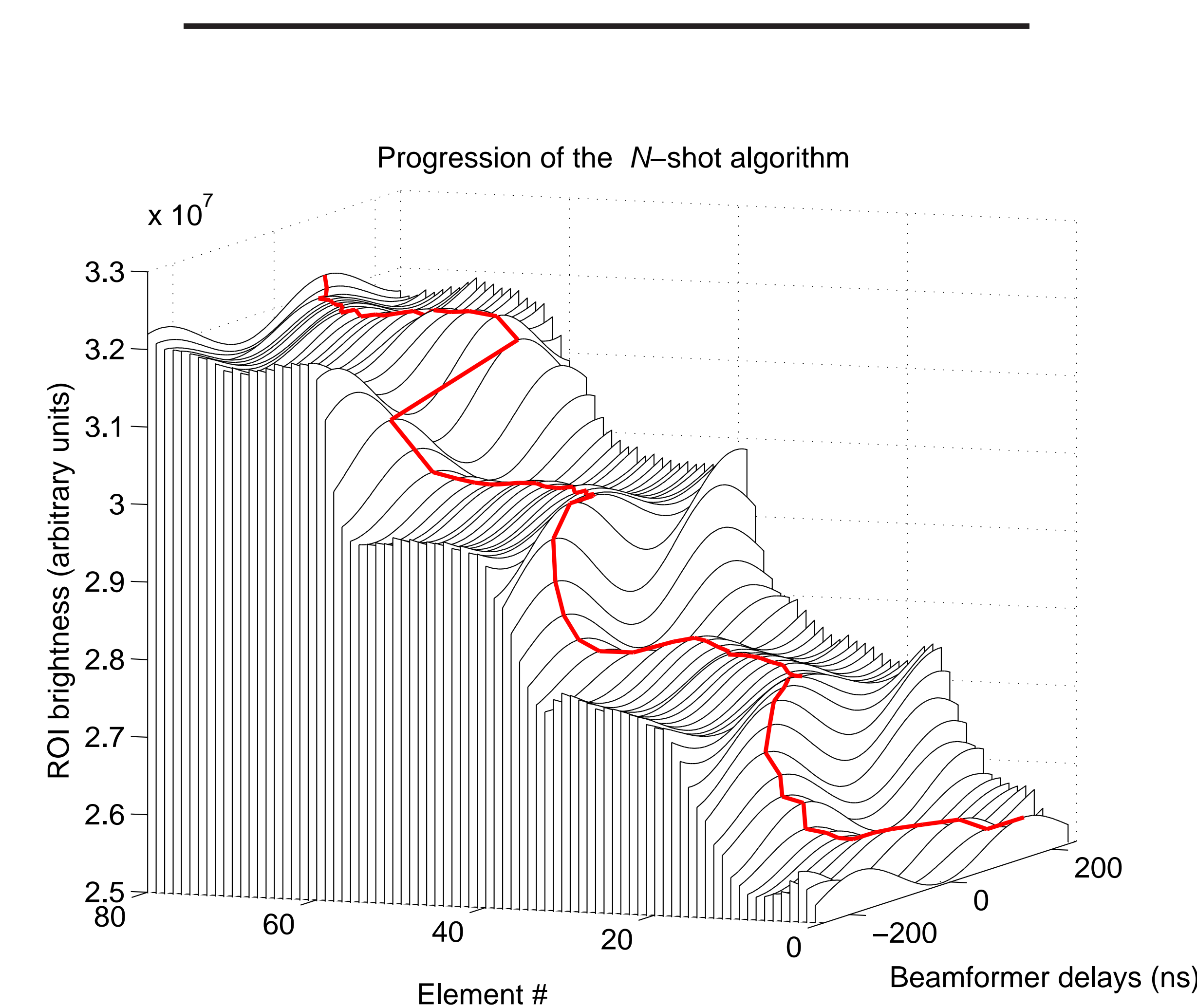
Compared to the algorithm in [2], this is **deterministic**, **faster**, and **more robust** in the presence of noise.

### 3 Three-shot algorithm

Use Equation (1) and angle addition properties to get the linear system

$$\begin{bmatrix} B_1(\tau_1) \\ B_2(\tau_2) \\ B_3(\tau_3) \end{bmatrix} = \begin{bmatrix} 1 & \cos(2\pi f_o \tau_1) & \sin(2\pi f_o \tau_1) \\ 1 & \cos(2\pi f_o \tau_2) & \sin(2\pi f_o \tau_2) \\ 1 & \cos(2\pi f_o \tau_3) & \sin(2\pi f_o \tau_3) \end{bmatrix} \begin{bmatrix} u \\ A \cos \phi \\ A \sin \phi \end{bmatrix} \quad (2)$$

which is easily solved for  $u$ ,  $A$ , and  $\phi$ . The optimal delay for this element is  $\phi/2\pi f_o$ ; the corresponding optimal brightness is  $u + A$ .



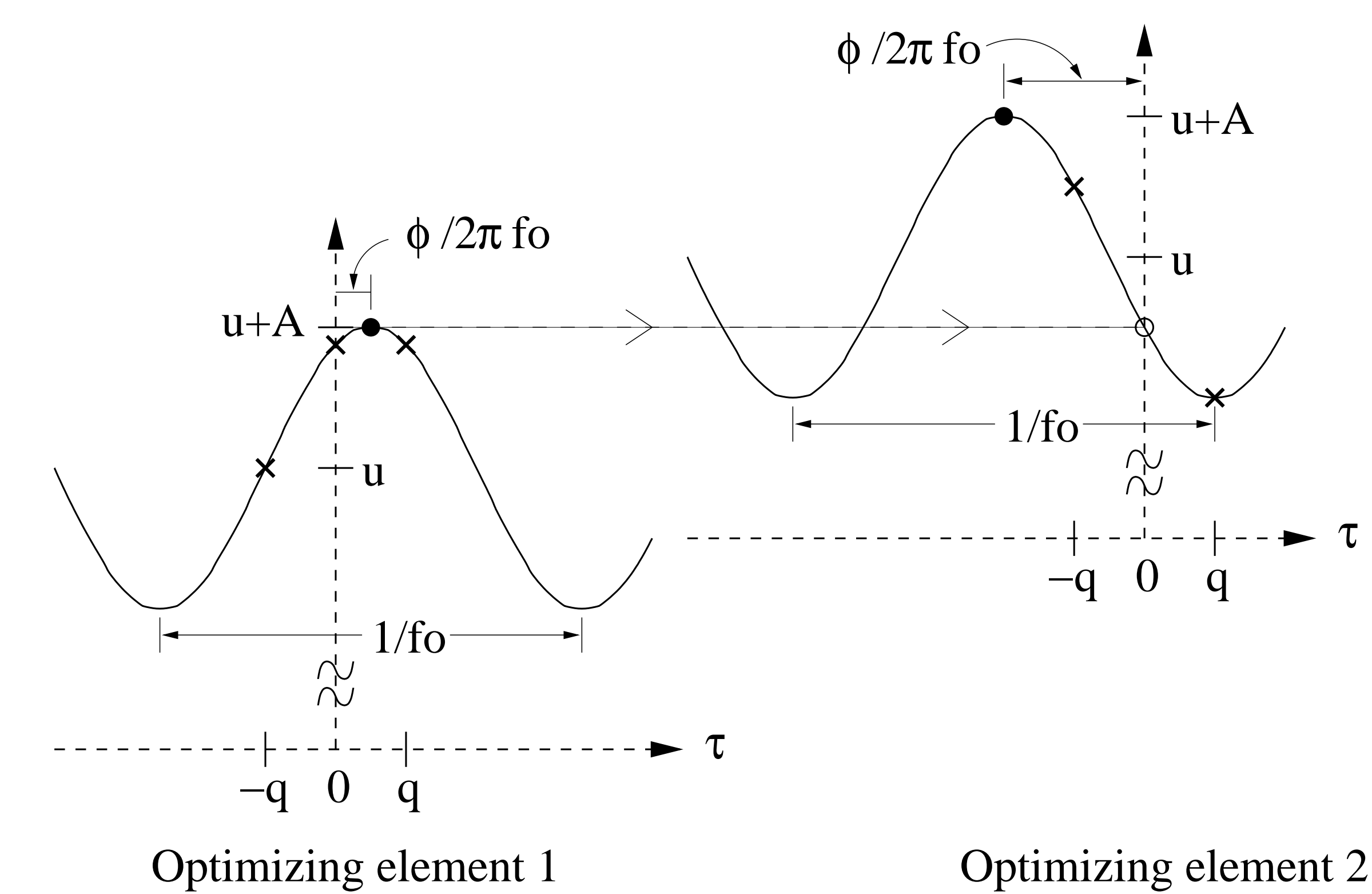
The *N*-shot algorithms estimate a sinusoidal brightness vs. delay dependence at each array element in turn. The displacement of the sinusoid peaks defines the estimated aberration profile (red line).

### 4 Two-shot algorithm

Start with the basic three-shot algorithm; suppose  $\tau_1 = 0$ , i.e. one shot is fired with the nominal delay.

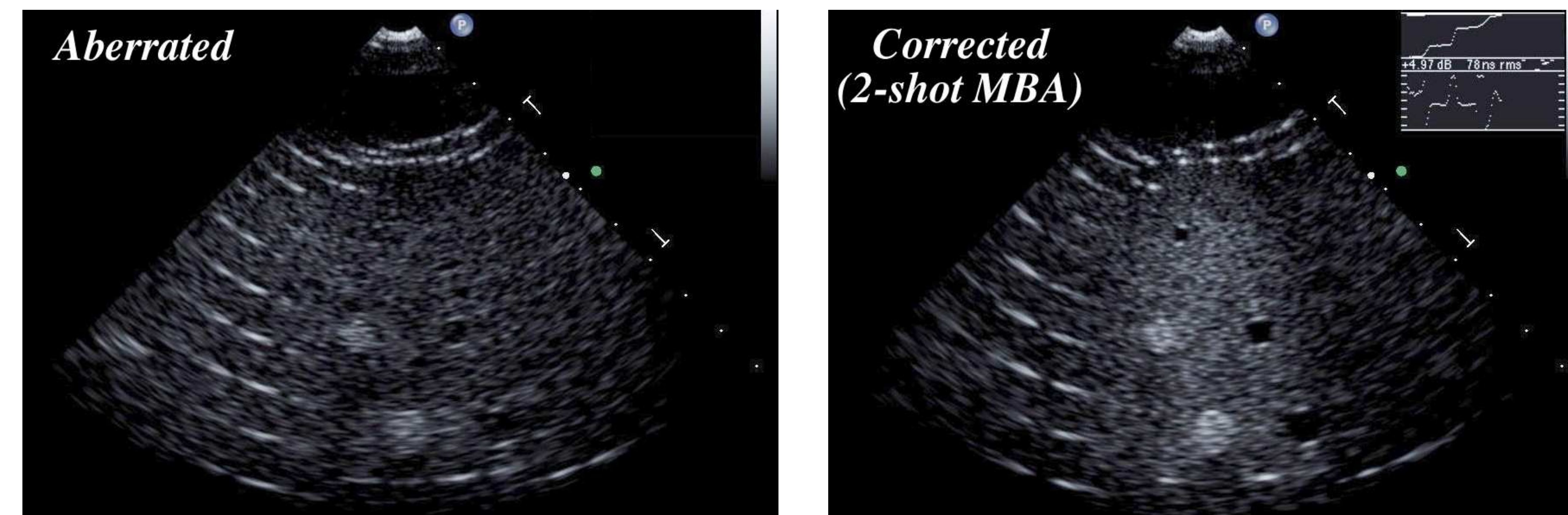
1. Fire ROI three times and find the optimal delay for the first element, as usual.
2. At the next element, *we already know* the result for  $\tau_1 = 0$ —the optimal brightness  $u + A$  from the previous solution!
3. Only need **two shots** for this and each subsequent element.

This reasoning can be illustrated graphically as follows:



*X*'s denote the positions on the [assumed] sinusoidal brightness profile sampled by the ROI firings (“shots”). An empty circle represents the “virtual” shot which relies on the previous element's delay solution.

### 5 Experimental results



The two-shot algorithm was used to correct this image of a phantom viewed through a silicone rubber aberrator (Figure 8 in [3]).

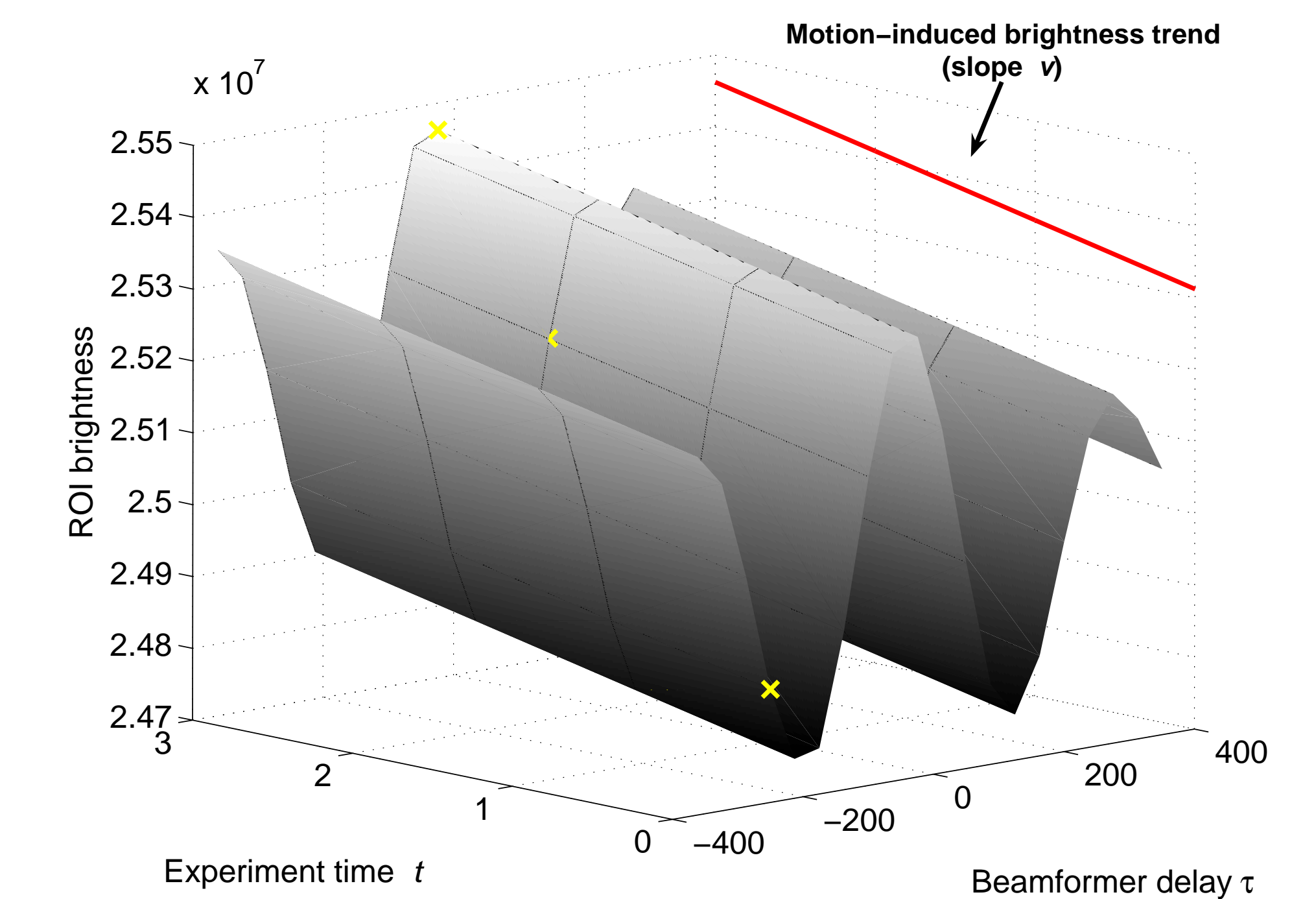
### 6 Motion-compensated algorithm

Tissue motion between shots could ruin the solution *in vivo*. (Motion-induced brightness changes between elements are harmless, at least for three-shot.)

To first order, model this as a brightness ramp over experiment time  $t$  (not to be confused with the beamformer delay  $\tau$ ):

$$B(\tau; t) = u + vt + A \cos(2\pi f_o \tau - \phi) \quad (3)$$

The modeled brightness as a function of beamformer delay and time is shown in the figure below, where the yellow X's represent the locations on the surface sampled by the four shots (one is hidden):



Now four shots are required to solve for the four unknowns:

$$\begin{bmatrix} B_1(\tau_1; t_1) \\ B_2(\tau_2; t_2) \\ B_3(\tau_3; t_3) \\ B_4(\tau_4; t_4) \end{bmatrix} = \begin{bmatrix} 1 & t_1 & \cos(2\pi f_o \tau_1) & \sin(2\pi f_o \tau_1) \\ 1 & t_2 & \cos(2\pi f_o \tau_2) & \sin(2\pi f_o \tau_2) \\ 1 & t_3 & \cos(2\pi f_o \tau_3) & \sin(2\pi f_o \tau_3) \\ 1 & t_4 & \cos(2\pi f_o \tau_4) & \sin(2\pi f_o \tau_4) \end{bmatrix} \begin{bmatrix} u \\ v \\ A \cos \phi \\ A \sin \phi \end{bmatrix} \quad (4)$$

Note:

- Can derive a “three-shot motion-compensated” variant with reasoning similar to two-shot.
- Could fire more shots and solve higher-order models.
- Could use extra shots for general redundancy and solve in the least-squares sense.

### 7 Conclusions

- The *N*-shot algorithms exploit the sinusoidal brightness vs. delay property to deliver improved speed and noise immunity.
- The optimal delays can be found with just two firings of the ROI lines per array element (the two-shot algorithm); another firing can estimate and compensate for first-order motion effects.
- In experiments imaging a phantom through an RTV silicone aberrator, the two-shot algorithm has proven both efficient and robust.

### References

- [1] R. A. Muller and A. Buffington, “Real-time correction of atmospherically degraded telescope images through image sharpening,” *Journal of the Optical Society of America*, vol. 64, no. 9, pp. 1200–1209, 1974.
- [2] L. Nock, G. E. Trahey, and S. W. Smith, “Phase aberration correction in medical ultrasound using speckle brightness as a quality factor,” *Journal of the Acoustical Society of America*, vol. 85, no. 5, pp. 1819–1833, May 1989.
- [3] M. A. Haun, D. L. Jones, and W. D. O'Brien, Jr., “Overdetermined least-squares aberration estimates,” *IEEE Transactions on Medical Imaging*, vol. 23, no. 10, pp. 1205–1220, October 2004.